EE 508
Lecture 22

Sensitivity Functions

- Comparison of Circuits
- Predistortion and Calibration

Review Correction from last time

Theorem: If all op amps in a filter are ideal, then ω_o, Q, BW, all band edges, and all poles and zeros are homogeneous of order 0 in the impedances.

Theorem: If all op amps in a filter are ideal and if T(s) is a dimensionless transfer function, T(s), $T(j\omega)$, $|T(j\omega)|$, $\angle T(j\omega)$, are homogeneous of order 0 in the impedances

Review from last time

Bilinear Property of Electrical Networks

Theorem: Let x be any component or Op Amp time constant (1^{st} order Op Amp model) of any linear active network employing a finite number of amplifiers and lumped passive components. Any transfer function of the network can be expressed in the form

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

where N_0, N_1, D_0, and D_1 are polynomials in s that are not dependent upon x

A function that can be expressed as given above is said to be a bilinear function in the variable x and this is termed a bilateral property of electrical networks.

The bilinear relationship is useful for

1. Checking for possible errors in an analysis
2. Pole sensitivity analysis

Root Sensitivities

Consider expressing T(s) as a bilinear fraction in x

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}=\frac{N(s)}{D(s)}$$

Theorem: If z_i is any simple zero and/or p_i is any simple pole of T(s), then

$$S_x^{z_i}=\left(\frac{x}{z_i}\right)\left(\frac{-N_1(z_i)}{\frac{dN(z_i)}{dz_i}}\right) \quad \text{and} \quad S_x^{p_i}=\left(\frac{x}{p_i}\right)\left(\frac{-D_1(p_i)}{\frac{dD(p_i)}{dp_i}}\right)$$

Note: Do not need to find expressions for the poles or the zeros to find the pole and zero sensitivities !

Note: Do need the poles or zeros but they will generally be known by design

Note: Will make minor modifications for extreme values for x (i.e. τ for op amps)

Root Sensitivities

Theorem: If p_i is any simple pole of T(s), then

$$S_x^{p_i} = \left(\frac{x}{p_i}\right)\left(\frac{-D_1(p_i)}{\frac{dD(p_i)}{dp_i}}\right)$$

Proof (similar argument for the zeros)

$$D(s) = D_0(s) + xD_1(s)$$

By definition of a pole,

$$D(p_i) = 0$$

$$\therefore \quad D(p_i) = D_0(p_i) + xD_1(p_i) = 0$$

Root Sensitivities

$$\therefore \qquad D(p_i) = D_0(p_i) + xD_1(p_i)$$

Differentiating this expression implicitly WRT x, we obtain

$$\frac{\partial D_0(p_i)}{\partial p_i}\frac{\partial p_i}{\partial x} + \left[x\frac{\partial D_1(p_i)}{\partial p_i}\frac{\partial p_i}{\partial x} + D_1(p_i)\right] = 0$$

Re-grouping, obtain

$$\frac{\partial p_i}{\partial x}\left[\frac{\partial D_0(p_i)}{\partial p_i} + x\frac{\partial D_1(p_i)}{\partial p_i}\right] = -D_1(p_i)$$

But term in brackets is derivative of $D(p_i)$ wrt p_i, thus

$$\frac{\partial p_i}{\partial x} = -\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

Root Sensitivities

$$\frac{\partial p_i}{\partial x} = -\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

Finally, from the definition of sensitivity,

$$S_x^{p_i} = \frac{x}{p_i}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{p_i}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

Root Sensitivities

$$S_x^{p_i} = \frac{x}{p_i}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{p_i}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

Observation: Although the sensitivity expression is readily obtainable, direction information about the pole movement is obscured because the derivative is multiplied by the quantity p_i which is often complex. Usually will use either

$$\mathcal{S}_x^{p_i} = \frac{\partial p_i}{\partial x}$$

or

$$\tilde{S}_x^{p_i} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

which preserve direction information when working with pole or zero sensitivity analysis.

Root Sensitivities

Summary: Pole (or zero) locations due to component variations can be approximated with simple analytical calculations without obtaining parametric expressions for the poles (or zeros).

$$p_i \simeq p_i\Big|_{\substack{\text{Ideal}\\\text{Components}}} + \Delta p_i$$

where

$$\Delta p_i \simeq \Delta x \bullet \mathcal{S}_x^{p_i}$$

$$\mathcal{S}_x^{p_i} = -\frac{D_1(p_i)}{\left.\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)\right|_{p_{iN}}}$$

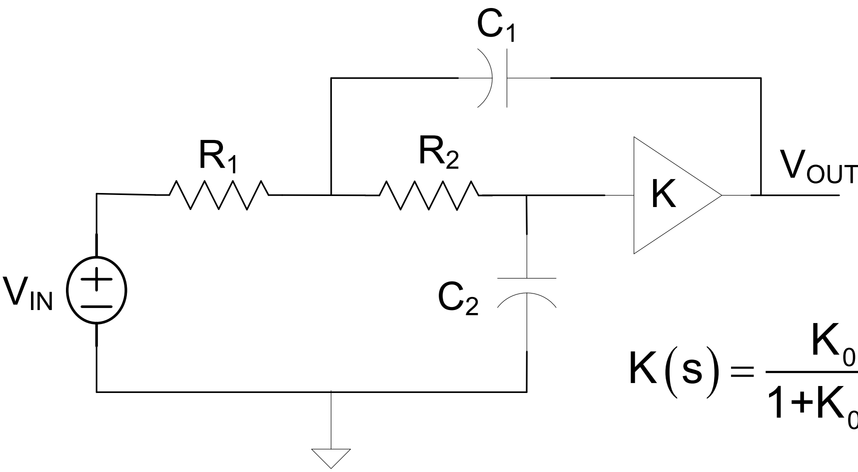
and

$$D(s) = D_0(s) + x \bullet D_1(s)$$

Alternately,

$$\Delta p_i \simeq \left(|p_i| \frac{\Delta x}{x}\right) \tilde{S}_x^{p_i}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

$$T(s)=\frac{\dfrac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{(1\text{-}K_0)}{R_2C_2}\right]+\dfrac{1}{R_1R_2C_1C_2}+K_0\tau s\left(s^2+s\left[\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}\right]+\dfrac{1}{R_1R_2C_1C_2}\right)}$$

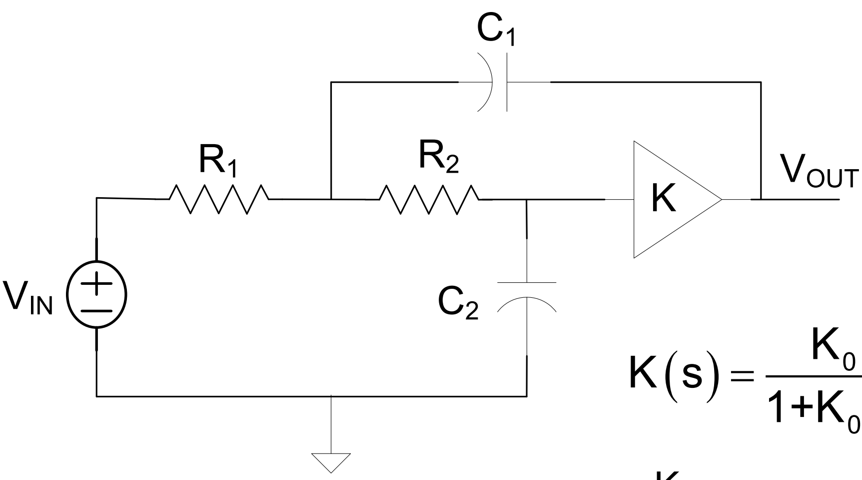
write in bilinear form

$$T(s)=\frac{\dfrac{K_0}{R_2C_1C_2}}{\left(s\dfrac{1}{C_1}+\dfrac{1}{R_2C_1C_2}+K_0\tau s\left(s\dfrac{1}{C_1}\right)+\dfrac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{(1\text{-}K_0)}{R_2C_2}+K_0\tau s\left(s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}\right]\right)\right]\right]}$$

evaluate at τ=0

$$T(s)=\frac{\dfrac{K_0}{R_2C_1C_2}}{\left(s\dfrac{1}{C_1}+\dfrac{1}{R_2C_1C_2}+\dfrac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{(1\text{-}K_0)}{R_2C_2}\right]\right]}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

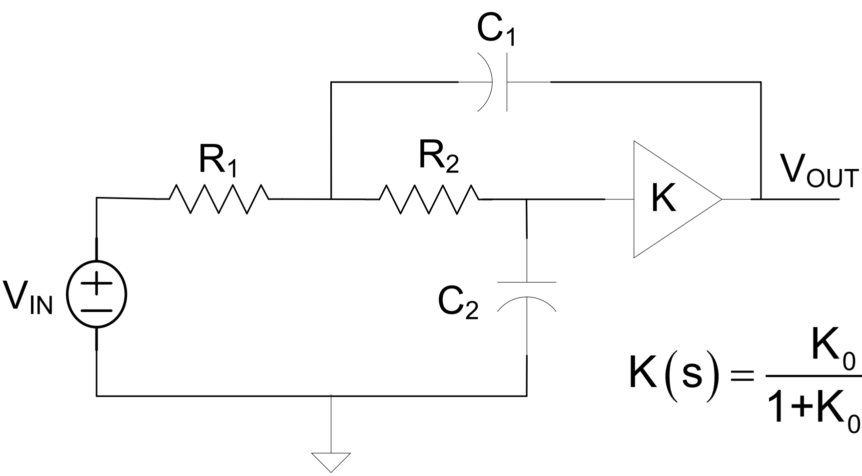
$$T(s)=\frac{\frac{K_0}{R_2C_1C_2}}{\left(s\frac{1}{C_1}+\frac{1}{R_2C_1C_2}+\frac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]\right]}$$

$$D_1(s)=s^2+s\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]$$

$$D(s)=\left(s\frac{1}{C_1}+\frac{1}{R_2C_1C_2}+\frac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]\right]=R_1\left(s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2\right)$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{1}{|p_i|}\right)\frac{p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$p^2+p\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}=0$$

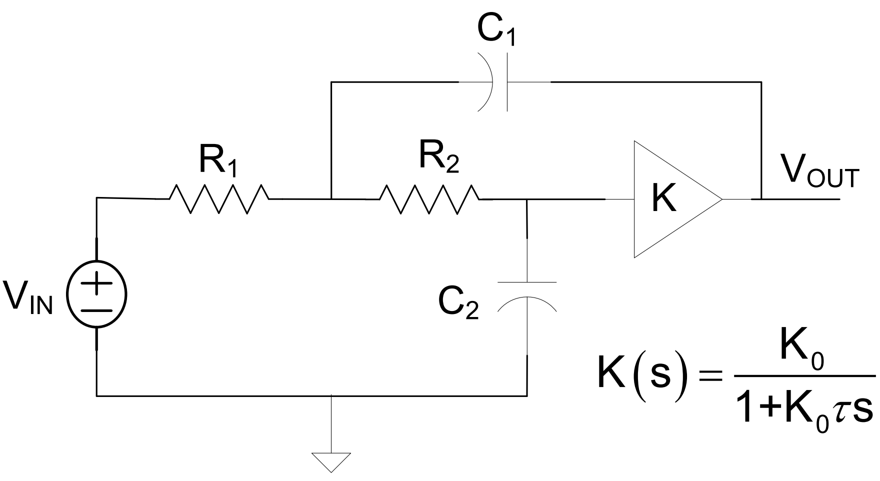
$$p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]=-\frac{1}{R_1R_2C_1C_2}-p\frac{1}{R_1C_1}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{1}{|p_i|}\right)\frac{p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{|p_i|}\right)\frac{\frac{1}{R_1R_2C_1C_2}+p\frac{1}{R_1C_1}}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{\omega_0}\right)\frac{\omega_0^2+p\frac{1}{R_1C_1}}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{\omega_0}\right)\frac{\omega_0^2+p\dfrac{1}{R_1C_1}}{\left(2p_i+\dfrac{\omega_0}{Q}\right)}$$

For equal R, equal C $\quad \omega_0 = \dfrac{1}{RC}$

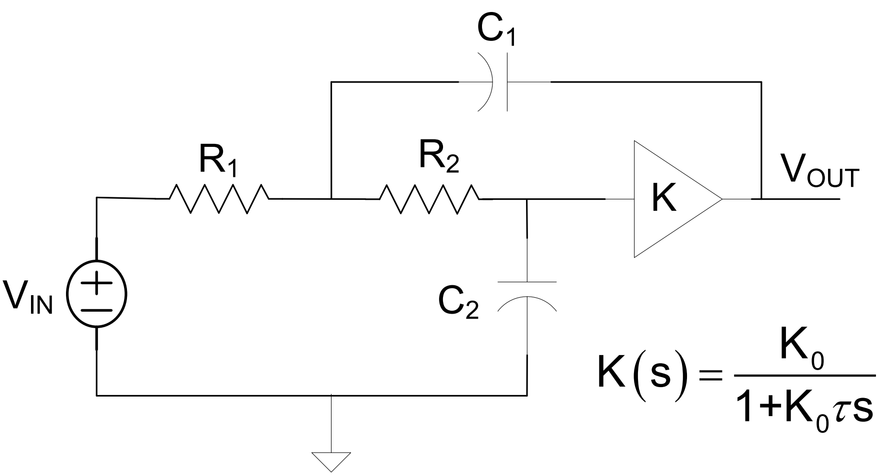
$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{\omega_0}\right)\frac{\omega_0^2+p\omega_0}{\left(2p_i+\dfrac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\frac{\omega_0+p}{\left(2p+\dfrac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{\omega_0 -\dfrac{\omega_0}{2Q}\pm\dfrac{\omega_0}{2Q}\sqrt{1-4Q^2}}{\pm\dfrac{\omega_0}{Q}\sqrt{1-4Q^2}}$$

$$\tilde{S}_{R_1}^{p}=\frac{Q-\dfrac{1}{2}\pm\dfrac{1}{2}\sqrt{1-4Q^2}}{\pm\sqrt{1-4Q^2}}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}$$

For equal R, equal C

$$\tilde{S}_{R_1}^{p}=\frac{Q-\frac{1}{2}\pm\frac{1}{2}\sqrt{1-4Q^2}}{\pm\sqrt{1-4Q^2}}$$

Note this contains magnitude and direction information

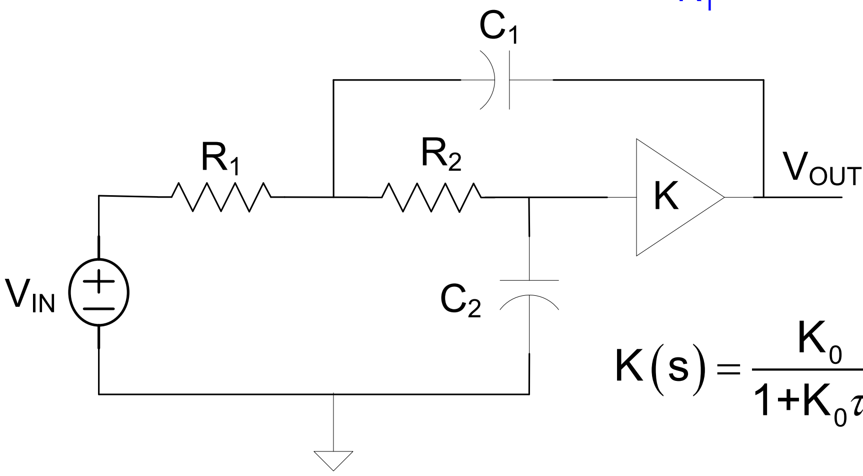
For high Q

$$\tilde{S}_{R_1}^{p}=\frac{Q\pm\frac{1}{2}\sqrt{-4Q^2}}{\pm\sqrt{-4Q^2}}=\frac{Q\pm jQ}{\pm j2Q}=\frac{1\pm j}{\pm j2}=\frac{j\pm 1}{\pm 2}=\frac{1}{2}\pm\frac{1}{2}j$$

$$\Delta p_i \cong |p_i|\tilde{S}_x^{p_i}\frac{\Delta x}{x}$$

$$\Delta p_i \cong \omega_0(0.5\pm 0.5j)\frac{\Delta R_1}{R_1}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$\tilde{S}_x^{p_i} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x}$$

For equal R, equal C

$$K(s) = \frac{K_0}{1+K_0\tau s}$$

For high Q

$$\Delta p_i \cong \omega_0 \left(0.5 \pm 0.5j\right)\frac{\Delta R_1}{R_1}$$

Could we have assumed equal R equal C before calculation?

No ! Analysis would not apply (not bilinear)

Results would obscure effects of variations in individual components

Was this a lot of work for such a simple result?

Yes ! But it is parametric and still only took maybe 20 minutes

But it needs to be done only once for this structure

Can do for each of the elements

What is the value of this result?

Understand how components affect performance of this circuit

Compare performance of different circuits for architecture selection

Transfer Function Sensitivities

$$S_x^{T(s)}\Big|_{s=j\omega} = S_x^{T(j\omega)}$$

$$S_x^{T(j\omega)} = S_x^{|T(j\omega)|} + j\theta S_x^{\theta}$$

where $\theta = \angle T(j\omega)$

$$S_x^{|T(j\omega)|} = \mathrm{Re}\left(S_x^{T(j\omega)}\right)$$

$$S_x^{\theta} = \frac{1}{\theta}\mathrm{Im}\left(S_x^{T(j\omega)}\right)$$

Transfer Function Sensitivities

If T(s) is expressed as $T(s) = \dfrac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \dfrac{N(s)}{D(s)}$

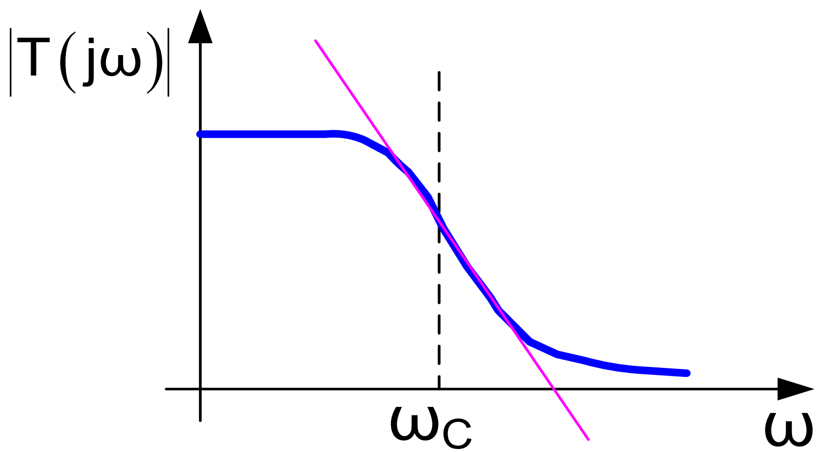
then
$$S_x^{T(s)} = \frac{\sum_{i=0}^{m} a_i s^i S_x^{a_i}}{N(s)} - \frac{\sum_{i=0}^{n} b_i s^i S_x^{b_i}}{D(s)}$$

If T(s) is expressed as $T(s) = \dfrac{N_0(s) + xN_1(s)}{D_0(s) + xD_1(s)}$

$$S_x^{T(s)} = \frac{x\left[D_0(s)N_1(s) - N_0(s)D_1(s)\right]}{\left(N_0(s) + xN_1(s)\right)\left(D_0(s) + xD_1(s)\right)}$$

Band-edge Sensitivities

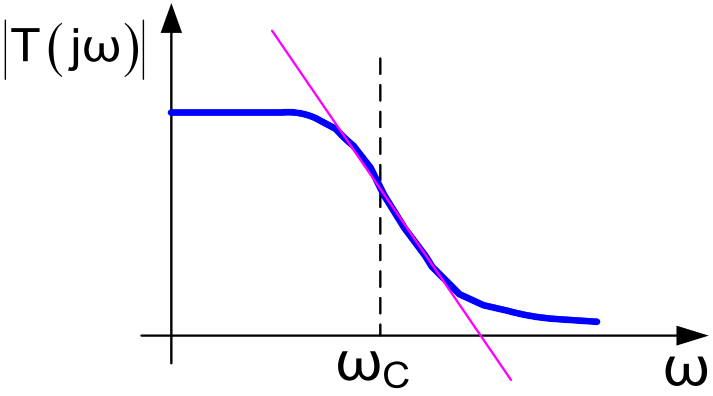
The band edge of a filter is often of interest. A closed-form expression for the band-edge of a filter may not be attainable and often the band-edges are distinct from the ω_0 of the poles. But the sensitivity of the band-edges to a parameter x is often of interest.



Want

$$S_x^{\omega_C} = \frac{\partial \omega_C}{\partial x} \bullet \frac{x}{\omega_C}$$

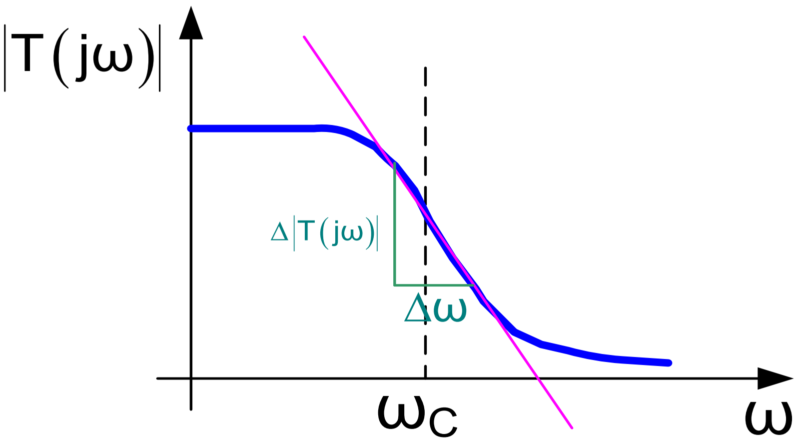
Band-edge Sensitivities



Theorem: The sensitivity of the band-edge of a filter is given by the expression

$$S_x^{\omega_C} = \frac{\left. S_x^{|T(j\omega)|} \right|_{\omega=\omega_C}}{\left. S_\omega^{|T(j\omega)|} \right|_{\omega=\omega_C}}$$

Band-edge Sensitivities

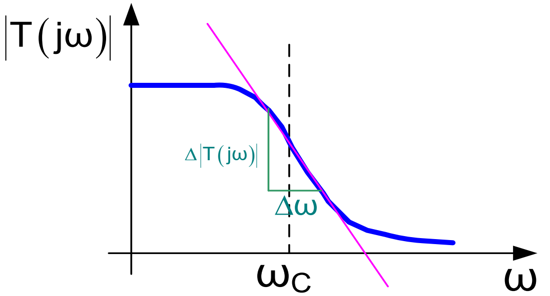


Proof:

Observe
$$\frac{\partial\left|T(j\omega)\right|}{\partial\omega} \cong \frac{\Delta\left|T(j\omega)\right|}{\Delta\omega}$$

$$\frac{\partial\left|T(j\omega)\right|}{\partial\omega} \cong \frac{\Delta\left|T(j\omega)\right|}{\Delta x} \bullet \frac{\Delta x}{\Delta\omega} \cong \frac{\dfrac{\partial\left|T(j\omega)\right|}{\partial x}}{\dfrac{\partial\omega}{\partial x}}$$

Band-edge Sensitivities



$$\frac{\partial |T(j\omega)|}{\partial \omega} \cong \frac{\Delta |T(j\omega)|}{\Delta x} \bullet \frac{\Delta x}{\Delta \omega} \cong \frac{\dfrac{\partial |T(j\omega)|}{\partial x}}{\dfrac{\partial \omega}{\partial x}}$$

$$\frac{\partial \omega}{\partial x} \cong \frac{\dfrac{\partial |T(j\omega)|}{\partial x}}{\dfrac{\partial |T(j\omega)|}{\partial \omega}}$$

$$\frac{\partial \omega}{\partial x} \cong \frac{\dfrac{\partial |T(j\omega)|}{\partial x} \bullet \dfrac{x}{|T(j\omega)|}}{\dfrac{\partial |T(j\omega)|}{\partial \omega} \bullet \dfrac{\omega}{|T(j\omega)|}} \left(\frac{\omega}{x}\right)$$

$$\frac{\partial \omega}{\partial x} \bullet \left(\frac{x}{\omega}\right) \cong \frac{\dfrac{\partial |T(j\omega)|}{\partial x} \bullet \dfrac{x}{|T(j\omega)|}}{\dfrac{\partial |T(j\omega)|}{\partial \omega} \bullet \dfrac{\omega}{|T(j\omega)|}}$$

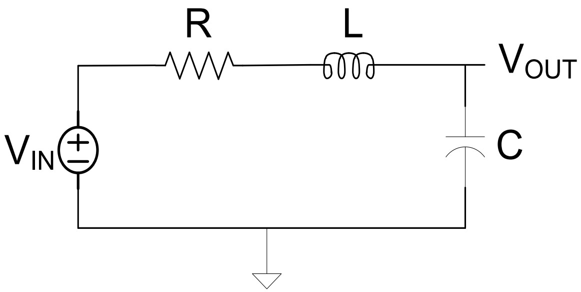
$$S_x^{\omega} = \frac{S_x^{|T(j\omega)|}}{S_\omega^{|T(j\omega)|}}$$

$$S_x^{\omega_C} = \frac{S_x^{|T(j\omega)|}\Big|_{\omega=\omega_C}}{S_\omega^{|T(j\omega)|}\Big|_{\omega=\omega_C}}$$

Sensitivity Comparisons

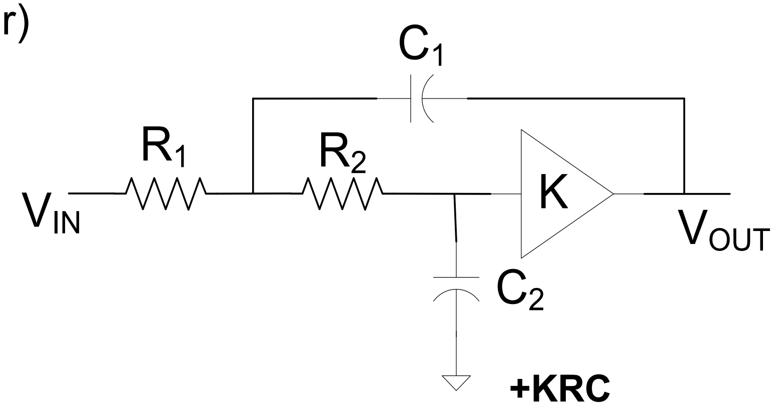
Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)



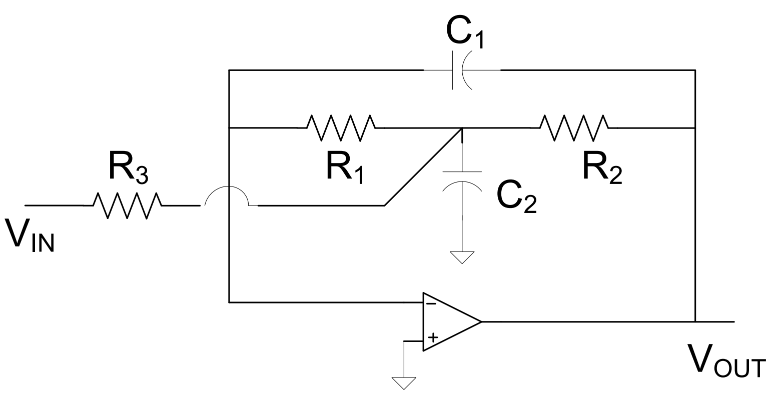
Passive RLC

(a)



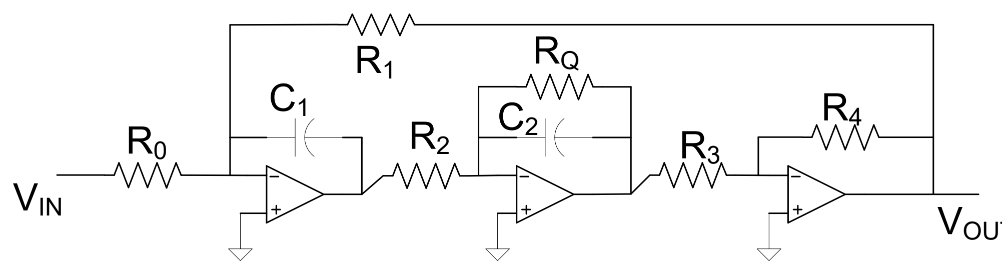
+KRC

(b)



Bridged-T Feedback

(c)



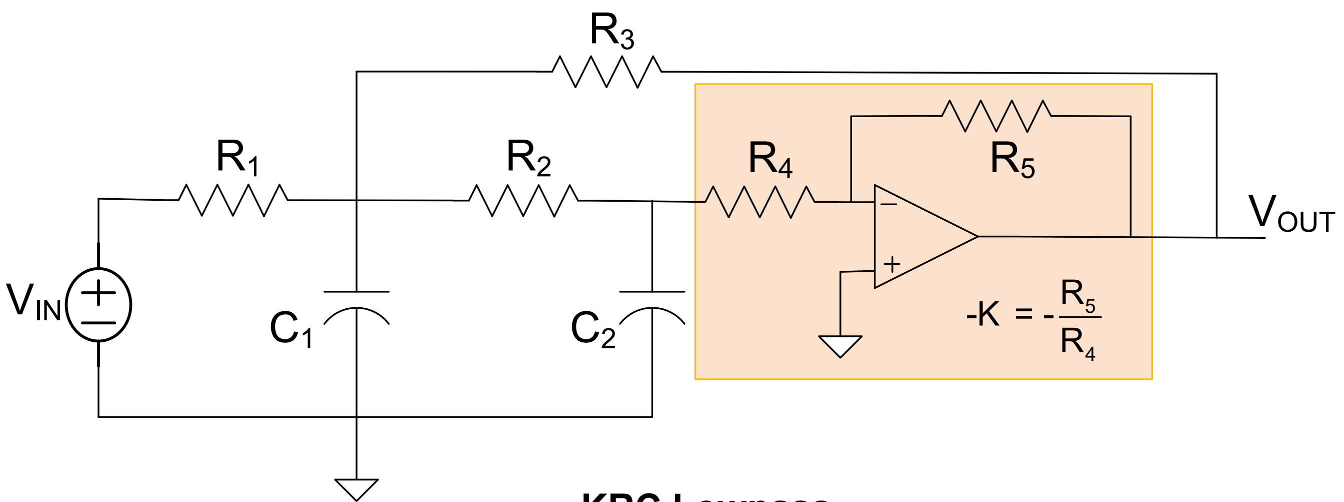
Two-Integrator Loop

(d)

Sensitivity Comparisons

Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)



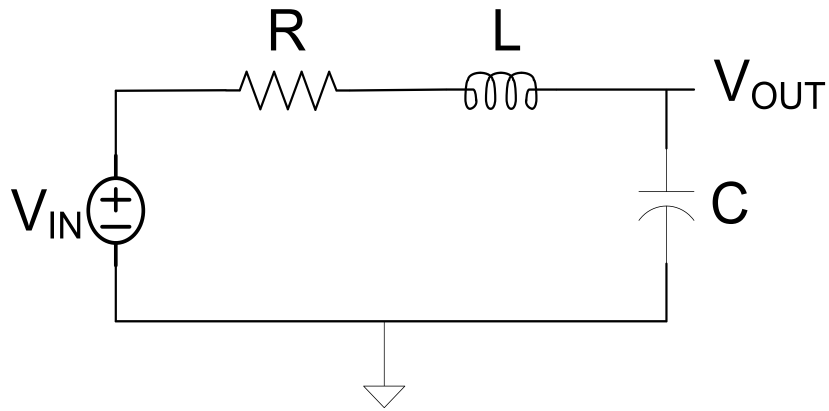
-KRC Lowpass

(e)

For all 5 structures, will have same transfer function within a gain factor

$$T(s) = \frac{K\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

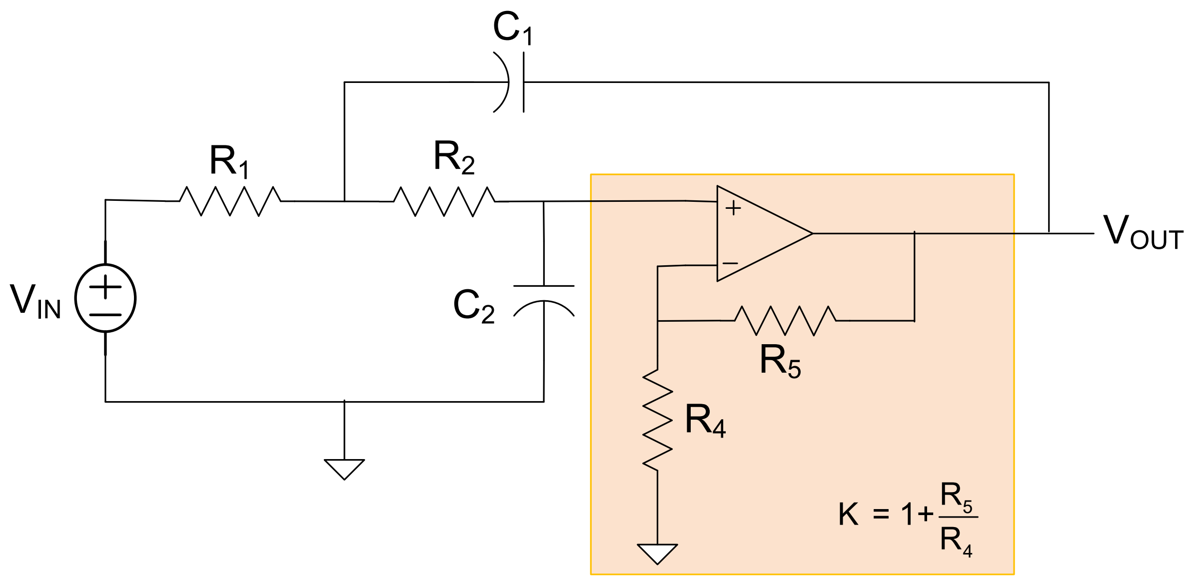
a) – Passive RLC



$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{1/LC}{s^2 + s\frac{R}{L} + 1/LC}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

b) + KRC (a Sallen and Key filter)



$$T(s) = \frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2 + s\left[\left(\dfrac{1}{\sqrt{R_1R_2C_1C_2}}\right)\left(\sqrt{\dfrac{R_1C_1}{R_2C_2}} + \sqrt{\dfrac{R_2C_2}{R_1C_1}} + \sqrt{\dfrac{R_1C_2}{R_2C_1}} - K\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right)\right] + \dfrac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\dfrac{R_1C_1}{R_2C_2}} + \sqrt{\dfrac{R_2C_2}{R_1C_1}} + \sqrt{\dfrac{R_1C_2}{R_2C_1}} - K\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right)}$$

Case b1 : Equal R, Equal C

$$R_1 = R_2 = R \qquad C_1 = C_2 = C$$

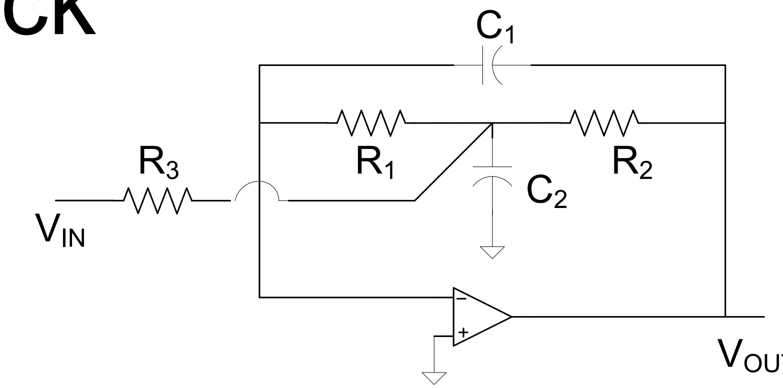
$$\omega_0 = \frac{1}{RC} \qquad K = 3 - \frac{1}{Q}$$

Case b2 : Equal R, K=1

$$R_1 = R_2 = R \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$$T(s) = \frac{K\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

c) Bridged T Feedback



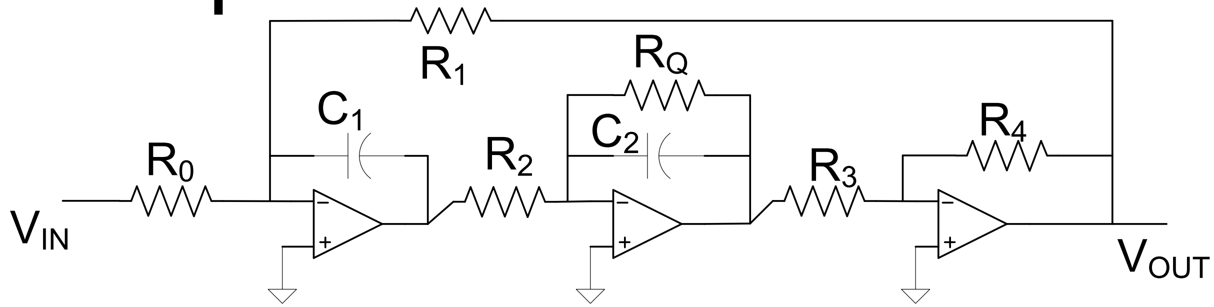
$$T(s) = \frac{\frac{1}{R_1R_3C_1C_2}}{s^2+s\left[\left(\sqrt{\frac{C_2}{C_1}}\right)\left(\frac{1}{\sqrt{R_1R_2C_1C_2}}\right)\left(\sqrt{\frac{R_1}{R_3}}+\sqrt{\frac{R_2}{R_1}}+\frac{\sqrt{R_1R_2}}{R_3}\right)\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}} \qquad Q = \frac{1}{\left(\sqrt{\frac{C_2}{C_1}}\right)\left(\sqrt{\frac{R_1}{R_3}}+\sqrt{\frac{R_2}{R_1}}+\frac{\sqrt{R_1R_2}}{R_3}\right)}$$

If $R_1=R_2=R_3=R$ and $C_2=9Q^2C_1$

$$T(s) = \frac{\frac{1}{9Q^2R^2C_1^2}}{s^2+s\left[\left(\frac{1}{3Q^2RC_1}\right)\right]+\frac{1}{9Q^2R^2C_1^2}}$$

d) 2 integrator loop



$$T(s) = -\frac{\frac{R_4}{R_3} \bullet \frac{1}{R_0 R_2 C_1 C_2}}{s^2 + s\left(\frac{1}{R_Q C_2}\right) + \frac{R_4}{R_3} \bullet \frac{1}{R_0 R_2 C_1 C_2}}$$

$$\omega_0 = \sqrt{\frac{R_4}{R_3} \bullet \frac{1}{R_0 R_2 C_1 C_2}}$$

$$Q = \frac{R_Q}{\sqrt{R_0 R_2}} \sqrt{\frac{C_2}{C_1}}$$

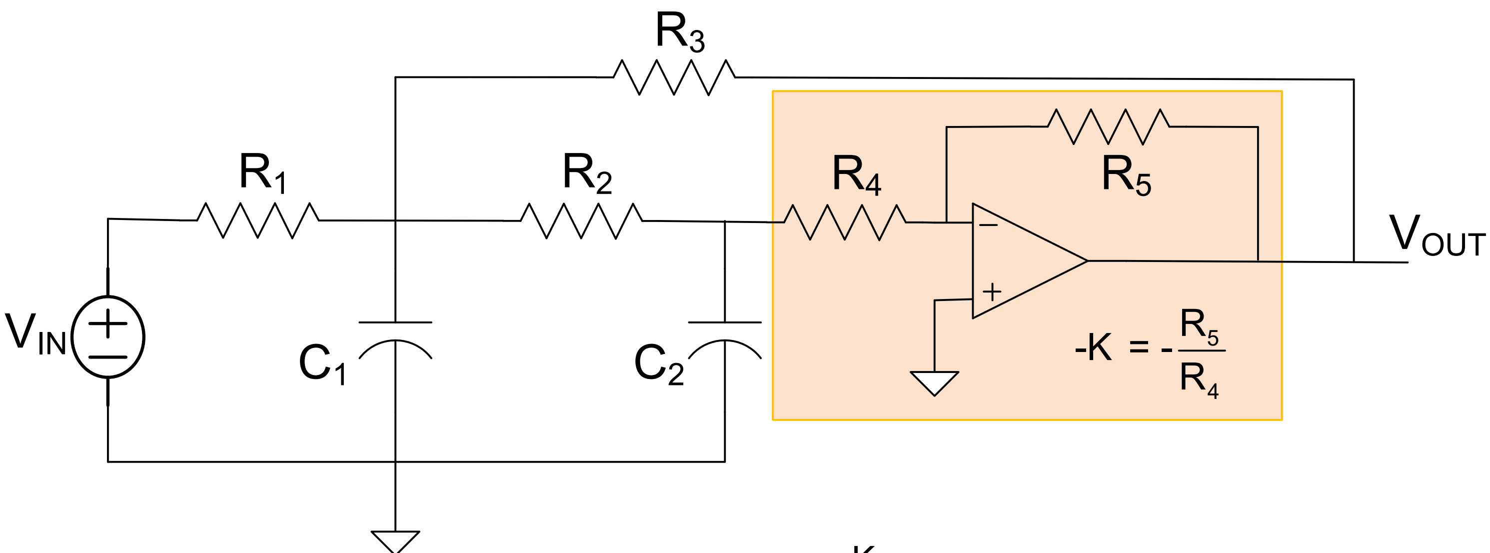
For: $R_0 = R_1 = R_2 = R$ $\quad C_1 = C_2 = C$ $\quad R_3 = R_4$

$$T(s) = -\frac{\frac{1}{R^2 C^2}}{s^2 + s\left(\frac{1}{R_Q C}\right) + \frac{1}{R^2 C^2}}$$

$R_Q = QR$

$$\omega_0 = \frac{1}{RC}$$

d) - KRC (a Sallen and Key filter)



$$T(s) = -\frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2+s\left[\left(1+\dfrac{R_1}{R_3}\right)\left(\dfrac{1}{R_1C_1}\right)+\left(1+\dfrac{C_2}{C_1}\right)\left(\dfrac{1}{R_2C_2}\right)+\left(\dfrac{1}{R_4C_2}\right)\right]+\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}}{\left(1+\dfrac{R_1}{R_3}\right)\left(\dfrac{1}{R_1C_1}\right)+\left(1+\dfrac{C_2}{C_1}\right)\left(\dfrac{1}{R_2C_2}\right)+\left(\dfrac{1}{R_4C_2}\right)}$$

Often $R_1=R_2=R_3=R_4=R$, $C_1=C_2=C$

$$Q = \frac{\sqrt{5+K_0}}{5}$$

How do these five circuits compare?

a) From a passive sensitivity viewpoint?

- If Q is small
- If Q is large

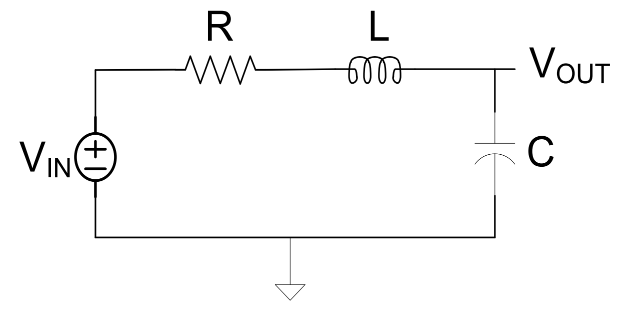
b) From an active sensitivity viewpoint?

- If Q is small
- If Q is large
- If $\tau\omega_0$ is large

Comparison: Calculate all ω_0 and Q sensitivities

Consider passive sensitivities first

a) – Passive RLC



$$S_R^{\omega_0} = 0$$

$$S_L^{\omega_0} = -\frac{1}{2}$$

$$S_C^{\omega_0} = -\frac{1}{2}$$

$$S_R^{Q} = -1$$

$$S_C^{Q} = -\frac{1}{2}$$

$$S_L^{Q} = \frac{1}{2}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Case b1 : +KRC Equal R, Equal C

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}} - K\sqrt{\frac{R_1C_1}{R_2C_2}}\right)}$$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_{K}^{\omega_0} = 0$$

$$S_{R_1}^{Q} = Q - \frac{1}{2}$$

$$S_{R_2}^{Q} = -Q + \frac{1}{2}$$

$$S_{C_1}^{Q} = 2Q - \frac{1}{2}$$

$$S_{C_2}^{Q} = -2Q + \frac{1}{2}$$

$$S_{K}^{Q} = 3Q - 1$$

$$Q = \frac{1}{3 - K}$$

$$\omega_0 = \frac{1}{RC}$$

If $Q_N = 10$, what happens to Q if R_1 increases by 1% ?

10% ?

$\frac{\Delta R_1}{R_1} = .01$

$$\frac{\Delta Q}{Q} \approx S^{Q}_{R_1} \cdot \frac{\Delta R_1}{R_1} \approx (Q - 1/2)(.01) = .095$$

$\therefore$ Q changes by 9.5%

$\frac{\Delta R_1}{R_1} = 0.1$

$$\frac{\Delta Q}{Q} \approx S^{Q}_{R_1} \cdot \frac{\Delta R_1}{R_1} = (9.5)(.1) = .95$$

$\therefore$ Q changes by 95%

Actual: 10 → 11.04 for $\frac{\Delta R}{R} = .01$

10 → 105 for $\frac{\Delta R}{R} = 0.1$

Case b2 : +KRC Equal R, K=1

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_1C_1}{R_2C_2}}+\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}-K\sqrt{\frac{R_1C_1}{R_2C_2}}\right)}$$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_K^{\omega_0} = 0$$

$$S_{R_1}^{Q} = 0$$

$$S_{R_2}^{Q} = 0$$

$$S_{C_1}^{Q} = \frac{1}{2}$$

$$S_{C_2}^{Q} = -\frac{1}{2}$$

$$S_K^{Q} = 2Q^2$$

$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

c) Bridged T Feedback

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{C_2}{C_1}}\right)\left(\sqrt{\frac{R_1}{R_3}}+\sqrt{\frac{R_2}{R_1}}+\frac{\sqrt{R_1R_2}}{R_3}\right)}$$

For $R_1=R_2=R_3=R$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_{R_3}^{\omega_0} = 0$$

$$S_{R_1}^{Q} = -\frac{1}{6}$$

$$S_{R_2}^{Q} = -\frac{1}{6}$$

$$S_{R_3}^{Q} = \frac{1}{3}$$

$$S_{C_1}^{Q} = -\frac{1}{2}$$

$$S_{C_2}^{Q} = \frac{1}{2}$$

$$\omega_0 = \frac{3Q}{RC_1}$$

$$Q = \frac{1}{3}\sqrt{\frac{C_1}{C_2}}$$

d) 2 integrator loop

$$\omega_0 = \sqrt{\frac{R_4}{R_3} \cdot \frac{1}{R_0 R_2 C_1 C_2}}$$

$$Q = \frac{R_Q}{\sqrt{R_0 R_2}} \sqrt{\frac{C_2}{C_1}}$$

For: $R_0 = R_1 = R_2 = R$ $\quad C_1 = C_2 = C$ $\quad R_3 = R_4$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \qquad S_{R_4}^{\omega_0} = \frac{1}{2}$$

$$S_{R_1}^{Q} = S_{R_2}^{Q} = S_{R_3}^{Q} = S_{C_1}^{Q} = -\frac{1}{2}$$

$$S_{R_4}^{Q} = S_{C_2}^{Q} = \frac{1}{2}$$

$$S_{R_Q}^{Q} = 1$$

$$S_{R_0}^{Q} = 0$$

$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{R_Q}{R}$$

d) -KRC passive sensitivities

$$\omega_0 = \sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}}{\left(1+\frac{R_1}{R_3}\right)\left(\frac{1}{R_1C_1}\right)+\left(1+\frac{C_2}{C_1}\right)\left(\frac{1}{R_2C_2}\right)+\left(\frac{1}{R_4C_2}\right)}$$

For $R_1=R_2=R_3=R_4=R$, $C_1=C_2=C$

$$Q = \frac{\sqrt{5+K_0}}{5} \qquad \omega_0 = \frac{\sqrt{5+K}}{R\,C}$$

$$S_{R_1}^{\omega_0} = -\frac{1}{25Q^2} \qquad S_{R_2}^{\omega_0} = -\frac{1}{2}+\frac{1}{25Q^2} \qquad S_{R_3}^{\omega_0} = -\frac{1}{2}+\frac{3}{50Q^2}$$

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \qquad S_{R_4}^{\omega_0} = -\frac{3}{50Q^2} \qquad S_{K}^{\omega_0} = \frac{1}{2}+\frac{1}{10Q^2}$$

$$S_{R_1}^{Q} = \frac{1}{5}-\frac{1}{25Q^2} \qquad S_{R_2}^{Q} = -\frac{1}{10}+\frac{1}{25Q^2} \qquad S_{R_3}^{Q} = -\frac{3}{10}+\frac{3}{50Q^2}$$

$$S_{R_4}^{Q} = \frac{1}{5}-\frac{3}{50Q^2} \qquad S_{C_2}^{Q} = -\frac{1}{10} \qquad S_{C_1}^{Q} = \frac{1}{10} \qquad S_{K}^{Q} = \frac{1}{2}-\frac{1}{10Q^2}$$

Passive Sensitivity Comparisons

	$\left|S_x^{\omega_0}\right|$	$\left|S_x^{Q}\right|$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Substantial Differences Between (or in) Architectures

How do active sensitivities compare?

$$S_{t}^{\omega_0} = ? \qquad S_{t}^{Q} = ?$$

Recall $S_{x}^{f} = \frac{\partial f}{\partial x} \frac{x}{f}$

so $\frac{\Delta f}{f} \simeq \frac{\Delta x}{x} S_{x}^{f}$

but if x is ideally 0, not useful

$$\mathcal{S}_{x}^{f} = \frac{\partial f}{\partial x}$$

$$\frac{\Delta f}{f} \simeq \mathcal{S}_{x}^{f} \frac{\Delta x}{f}$$

Where we are at with sensitivity analysis:

Considered a group of five second-order filters

Passive Sensitivity Analysis

- Closed form expressions were obtained for ω_0 and Q
- Tedious but straightforward calculations provided passive sensitivities directly from the closed form expressions ???

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

If we consider higher-order filters

Passive Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain for many useful structures

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

Need some better method for obtaining sensitivities when closed-form expressions are difficult or impractical to obtain or manipulate !!

Relationship between pole sensitivities and ω_0 and Q sensitivities

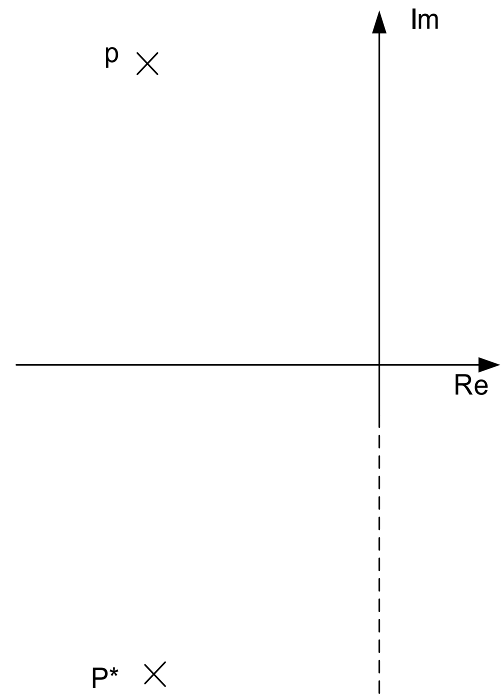
$p = -\alpha + j\beta$

$D_2(s) = (s-p)(s-p^*)$

$D_2(s) = (s+\alpha-j\beta)(s+\alpha+j\beta)$

$D_2(s) = s^2 + s(2\alpha) + (\alpha^2+\beta^2)$

$$D_2(s) = s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$



Relationship between active pole sensitivities and ω_0 and Q sensitivities

Define $D(s)=D_0(s)+\tau D_1(s)$ (from bilinear form of T(s))

Recall: $$\mathcal{S}_\tau^p = \frac{-D_1(p)}{\left.\frac{\partial D(s)}{\partial s}\right|_{s=p,\tau=0}}$$

Theorem: $$\Delta p \cong \tau \mathcal{S}_\tau^p$$

Theorem: $$\Delta\alpha \cong \tau \operatorname{Re}\left(\mathcal{S}_\tau^p\right)$$
$$\Delta\beta \cong \tau \operatorname{Im}\left(\mathcal{S}_\tau^p\right)$$

Theorem:

$$\frac{\Delta\omega_0}{\omega_0} \cong \frac{1}{2Q}\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\,\frac{\Delta\beta}{\omega_0}$$

$$\frac{\Delta Q}{Q} \cong -2Q\left(1-\frac{1}{4Q^2}\right)\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\,\frac{\Delta\beta}{\omega_0}$$

Claim: These theorems, with straightforward modification, also apply to other parameters (R, C, L, K, …) where, $D_0(s)$ and $D_1(s)$ will change since the parameter is different

Active Sensitivities

+KRC

Equal-R, Equal-C

$$\omega_o = \frac{1}{RC}, \qquad Q = \frac{1}{3-K_o}$$

$$\frac{V_o}{V_i} = \frac{\left(3-\frac{1}{Q}\right)\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{\left(3-\frac{1}{Q}\right)}{\mathrm{GB}} s(s^2 + s3\omega_o + \omega_o^2)} \qquad \left(\omega_o \ll \frac{\omega_t}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{1}{2Q}\left(3-\frac{1}{Q}\right)^2 \frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -\frac{1}{2}\left(3-\frac{1}{Q}\right)^2 \frac{\left(1-\frac{1}{2Q^2}\right)}{\sqrt{1-\frac{1}{4Q^2}}} \frac{\omega_o}{\mathrm{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong -\frac{1}{2}\left(3-\frac{1}{Q}\right)^2 \frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \cong \frac{1}{2}\left(3-\frac{1}{Q}\right)^2 \frac{\omega_o}{\mathrm{GB}}$$

Unity-gain, Equal-R

$$\omega_o = \frac{1}{R\sqrt{C_1C_2}}, \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$$\frac{V_o}{V_i} = \frac{\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{\mathrm{GB}}\left[s^2 + s\omega_o\left(2Q + \frac{1}{Q}\right) + \omega_o^2\right]} \qquad \left(\omega_o \ll \frac{\omega_t}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -Q \frac{\left(1-\frac{1}{2Q^2}\right)}{\sqrt{1-\frac{1}{4Q^2}}} \frac{\omega_o}{\mathrm{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong -Q\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \cong Q\frac{\omega_o}{\mathrm{GB}}$$

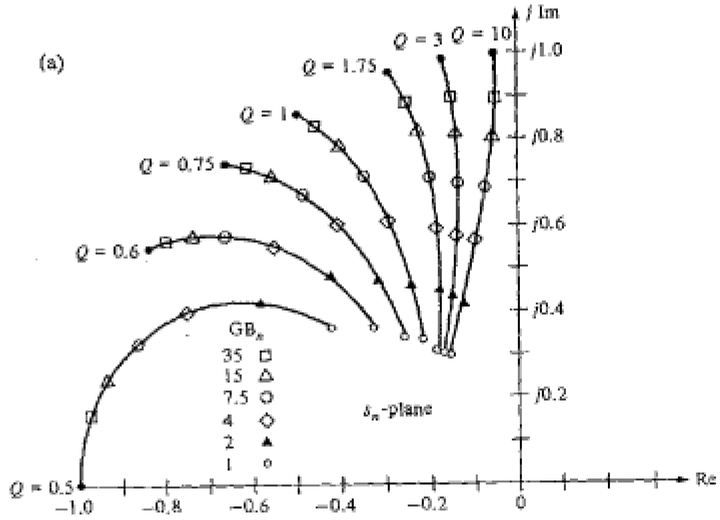
---

where

$$s_n = \frac{s}{\omega_o}, \qquad \mathrm{GB}_n = \frac{\mathrm{GB}}{\omega_o}.$$

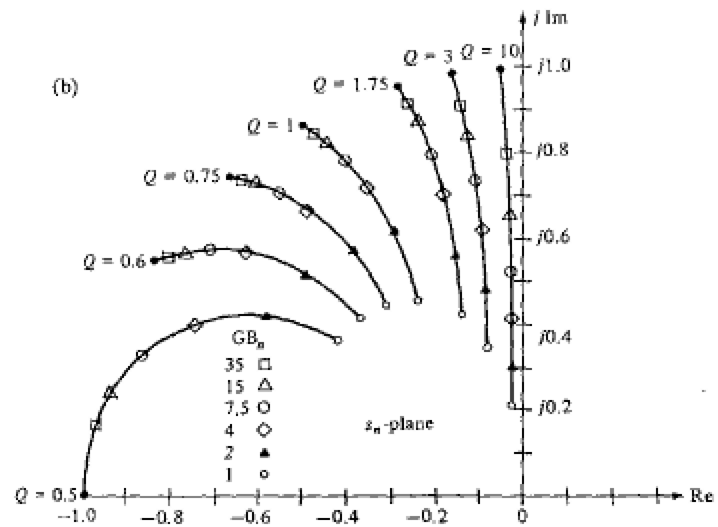
Active Sensitivities

+KRC



◀Fig. 10-5a Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(3+\frac{QGB_n}{3Q-1}\right) + s_n\left(1+\frac{GB_n}{3Q-1}\right) + \frac{QGB_n}{3Q-1} = 0$$ (Equal-R, equal-C)



◀Fig. 10-5b Plot of upper half plane root of

$$s_n^3 + s_n^2\left(2Q+\frac{1}{Q}+GB_n\right) + s_n\left(1+\frac{GB_n}{Q}\right) + GB_n = 0$$ (Unity-gain, equal-R)

Active Sensitivities

Bridged T Feedback

Table 10-3 Infinite-gain Realization
(see Fig. 10-10b)

Equal-R

$$\omega_o = \frac{1}{R\sqrt{C_1 C_2}}; \qquad Q = \frac{1}{3}\sqrt{\frac{C_1}{C_2}}$$

$$\frac{V_o}{V_i} = -\frac{\omega_o^2}{s^2 + s\dfrac{\omega_o}{Q} + \omega_o^2 + \dfrac{s}{\mathrm{GB}}\left[s^2 + s\omega_o\left(3Q + \dfrac{1}{Q}\right) + 2\omega_o^2\right]} \qquad \left(\omega_o \ll \frac{\omega_o}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \simeq \frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \simeq -\frac{1}{2}\frac{3Q - \dfrac{1}{Q}}{\sqrt{1 - \dfrac{1}{4Q^2}}}\frac{\omega_o}{\mathrm{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \simeq -\frac{3Q}{2}\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \simeq \frac{Q}{2}\frac{\omega_o}{\mathrm{GB}}$$

Active Sensitivities

Bridged T Feedback

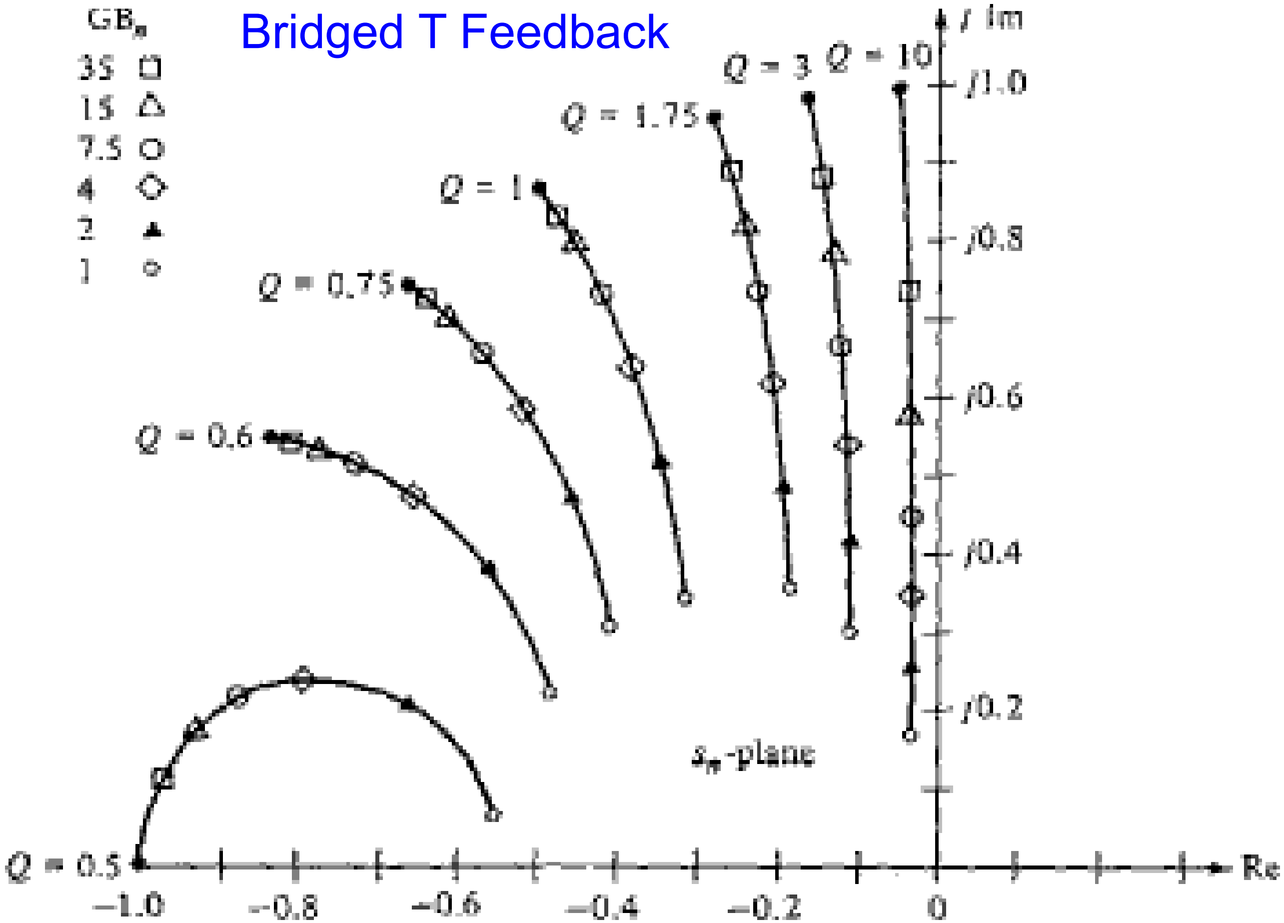


Fig. 10-12 Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(3Q + \frac{1}{Q} + \mathrm{GB}_n\right) + s_n\left(2 + \frac{\mathrm{GB}_n}{Q}\right) + \mathrm{GB}_n = 0$$

Active Sensitivities

Two integrator loop architecture

Equal-R (except R_Q) and Equal-C

$$\omega_o = \frac{1}{RC}, \qquad Q = \frac{R_Q}{R}$$

$$\frac{V_0}{V_i} \cong \frac{\omega_o^2\left(\frac{2}{\mathrm{GB}}s + 1\right)}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{1}{\mathrm{GB}}\left\{4s\left[s^2 + s\omega_o\left(\frac{1}{2} + \frac{1}{Q}\right) + \frac{\omega_o^2}{4Q}\right]\right\}} \qquad \left(\omega_o \ll \frac{\omega_a}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \simeq 2\left(1 + \frac{1}{4Q}\right)\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -\frac{\left(1 - \frac{1}{Q} - \frac{1}{4Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}}\frac{\omega_o}{\mathrm{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \simeq -\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \simeq 4Q\frac{\omega_o}{\mathrm{GB}}$$

Active Sensitivities

Two integrator loop architecture

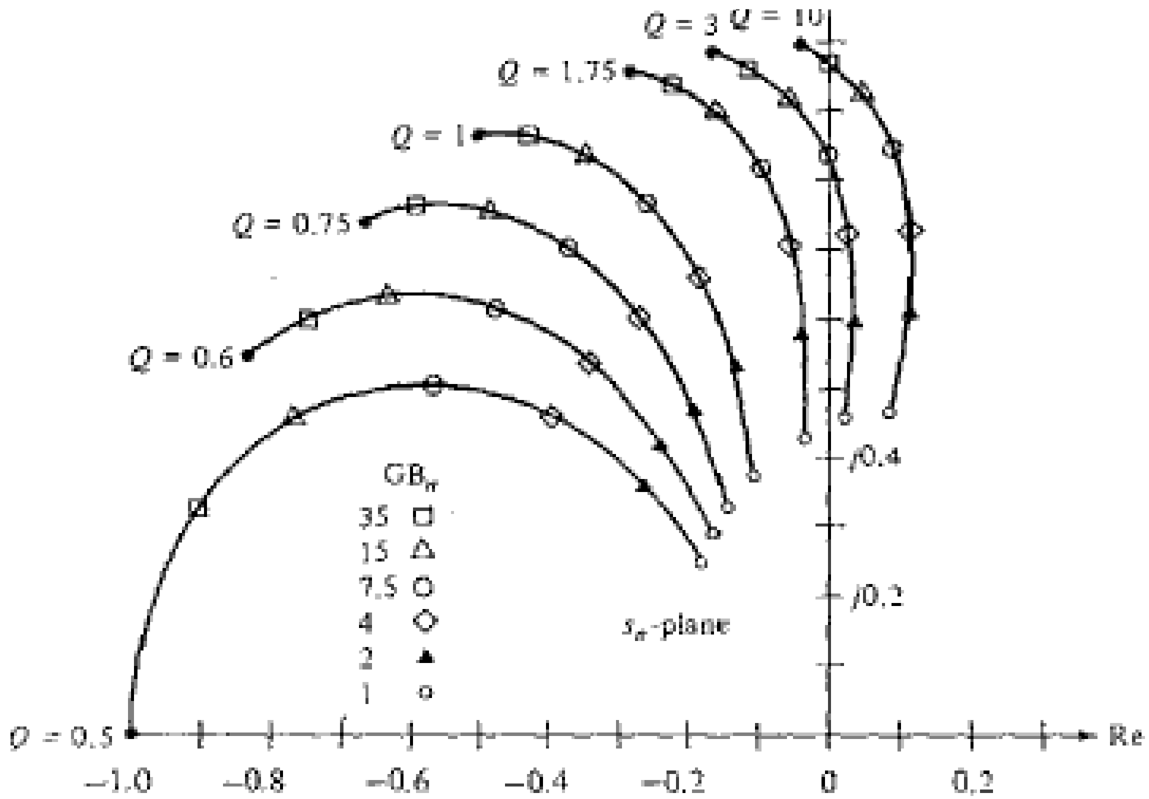


Fig. 10-17 Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(\frac{1}{2} + \frac{1}{Q} + \frac{GB_n}{4}\right) + s_n\frac{1}{4Q}\left(1 + GB_n\right) + \frac{GB_n}{4} = 0$$

Active Sensitivities

- KRC

Equal-R, Equal-C

$$\omega_o = \frac{\sqrt{5+K_0}}{RC}, \qquad Q = \frac{\sqrt{5+K_0}}{5}$$

$$\frac{V_o}{V_i} = -\frac{\omega_o^2\left(1-\frac{1}{5Q^2}\right)}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{GB}\left[s^2(25Q^2-4) + s\omega_o\left(20Q - \frac{3}{Q}\right) + \left(2-\frac{1}{5Q^2}\right)\omega_o^2\right]}$$

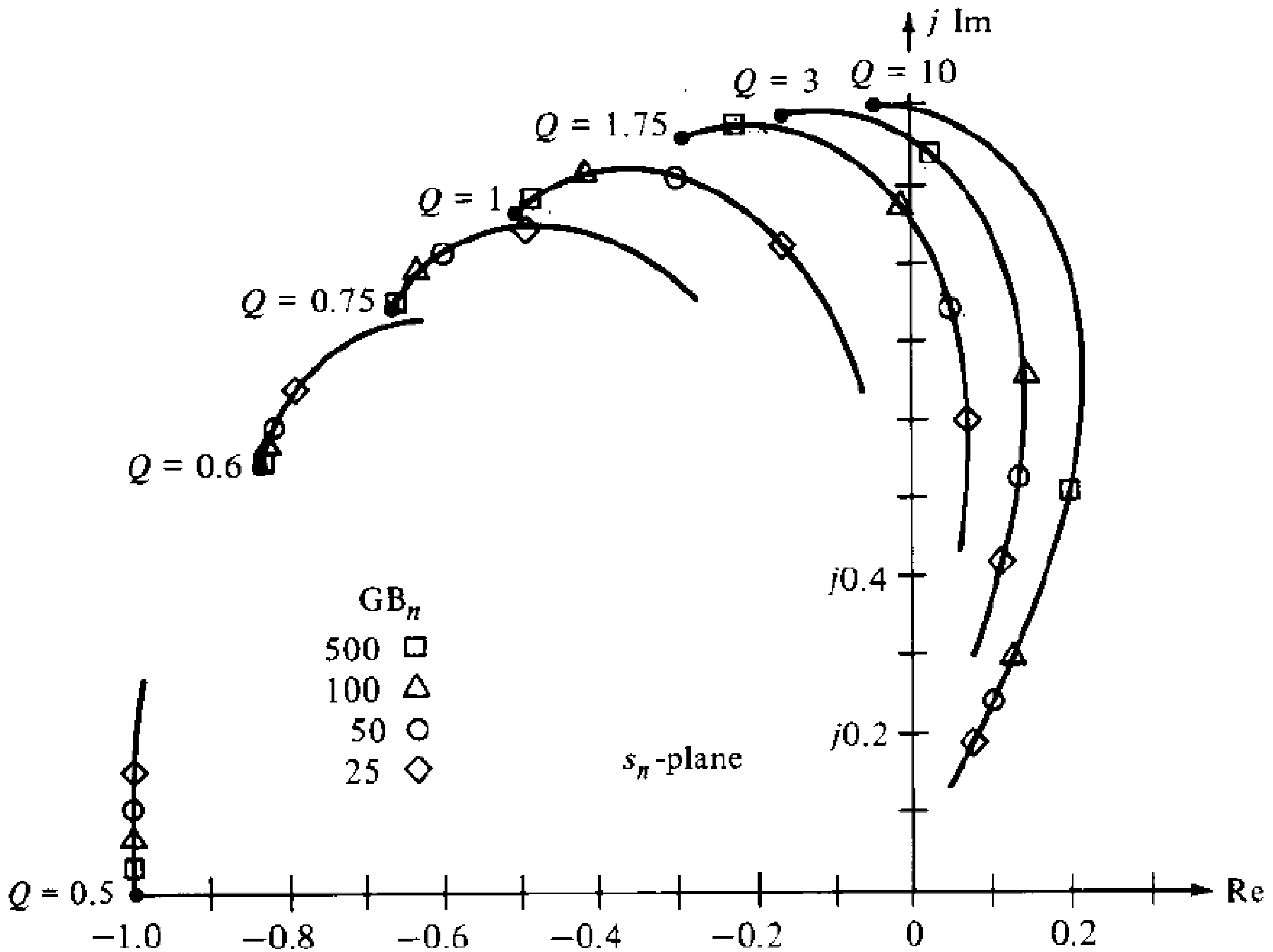
$$\left(\omega_a \ll \frac{\omega_o}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{25Q^2}{2}\left(1-\frac{1}{5Q^2}\right)\left(1-\frac{6}{25Q^2}\right)\frac{\omega_o}{GB}, \qquad \frac{\Delta\beta}{\omega_o} \cong \frac{35Q}{4}\frac{\left(1-\frac{1}{5Q^2}\right)\left(1-\frac{6}{35Q^2}\right)}{\sqrt{1-\frac{1}{4Q^2}}}\frac{\omega_o}{GB}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong \frac{5Q}{2}\left(1-\frac{1}{5Q^2}\right)\frac{\omega_o}{GB}, \qquad \frac{\Delta Q}{Q} \cong 25Q^3\left(1-\frac{1}{5Q^2}\right)\left(1-\frac{7}{5Q^2}\right)\frac{\omega_o}{GB}$$

Active Sensitivities

- KRC



Active Sensitivity Comparisons

	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
Passive RLC	NA	NA
+KRC		
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2 \tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2 \tau\omega_0$
Equal R, K=1 ($C_1=4Q^2C_2$)	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}Q\tau\omega_0$	$25Q^3\tau\omega_0$

Substantial Differences Between Architectures

Are these passive sensitivities acceptable?

	$\left\lvert S_x^{\omega_0} \right\rvert$	$\left\lvert S_x^{Q} \right\rvert$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Are these active sensitivities acceptable?

Active Sensitivity Comparisons

	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
Passive RLC		
+KRC		
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$
Equal R, K=1 ($C_1=4Q^2C_2$)	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}Q\tau\omega_0$	$25Q^3\tau\omega_0$

Are these sensitivities acceptable?

Passive Sensitivities:

$$\frac{\Delta\omega_0}{\omega_0} \cong S_x^{\omega_0} \frac{\Delta x}{x}$$

In integrated circuits, Δ R/R and Δ C/C due to process variations can be κ 30% or larger due to process variations

Many applications require $\Delta\omega_0/\omega_0 < .001$ or smaller and similar requirements on $\Delta Q/Q$

Even if sensitivity is around ½ or 1, variability is often orders of magnitude too large

Active Sensitivities:

All are proportional to $\tau\omega_0$

Some architectures much more sensitive than others

Can reduce $\tau\omega_0$ by making GB large but this is at the expense of increased power and even if power is not of concern, process presents fundamental limits on how large GB can be made

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained

Predistortion is generally used in integrated circuits to remove the bias associated with inadequate amplifier bandwidth

Predistortion does not help with process variations of passive components

Tedious process after fabrication since depends on individual components

Temperature dependence may not track

Difficult to maintain over time and temperature

Over-ordering will adversely affect performance

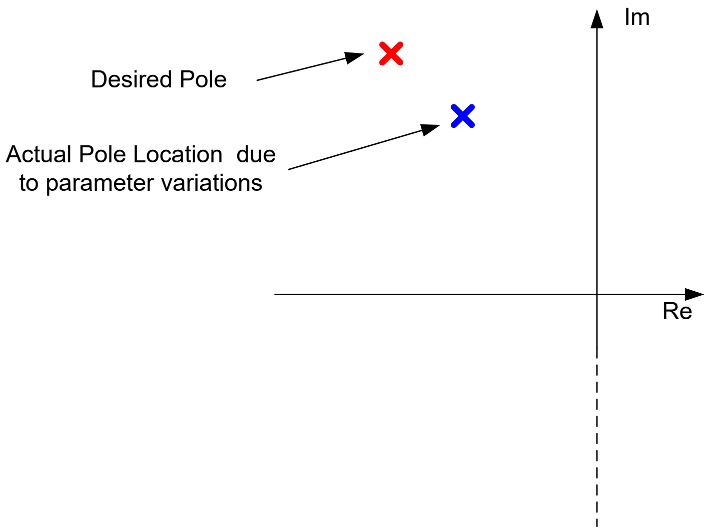
Seldom will predistortion alone be adequate to obtain acceptable performance

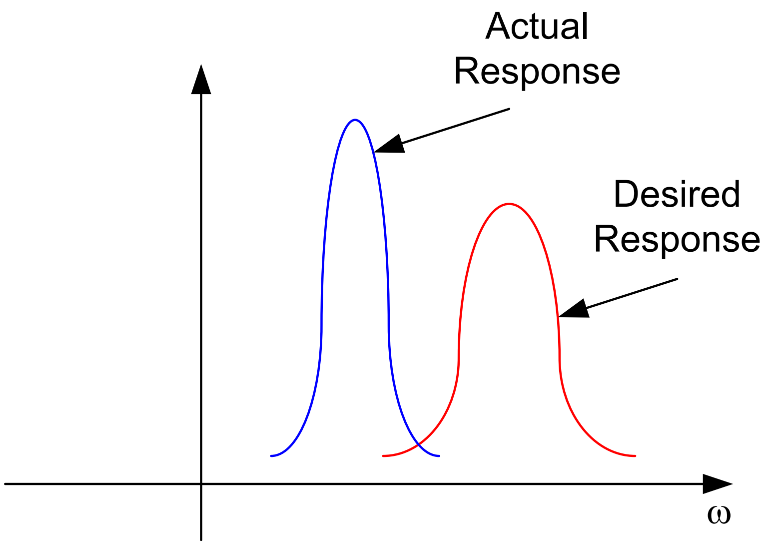
Bell Labs did to this in high-volume production (STAR Biquad)

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained



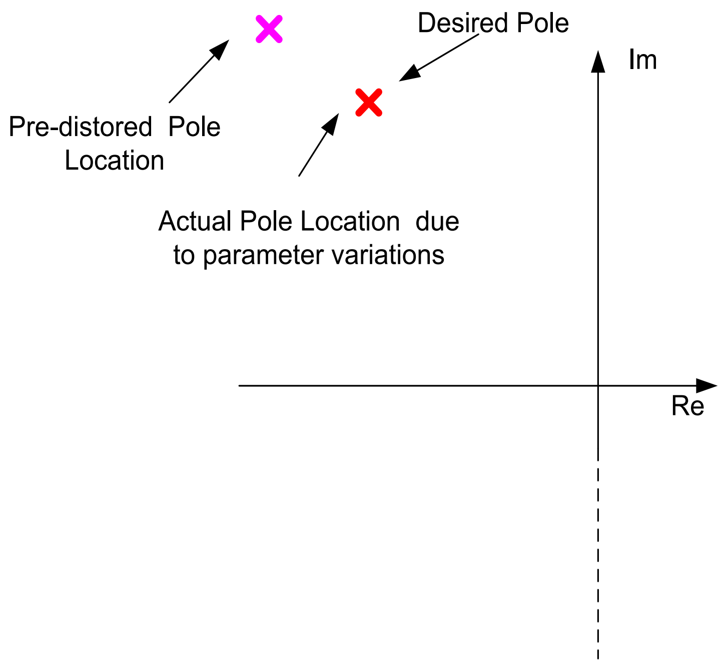


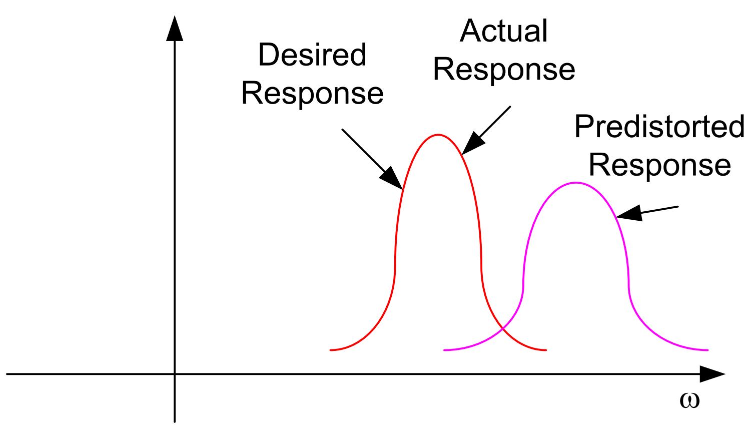
Pole shift due to parametric variations (e.g. inadequate GB)

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained



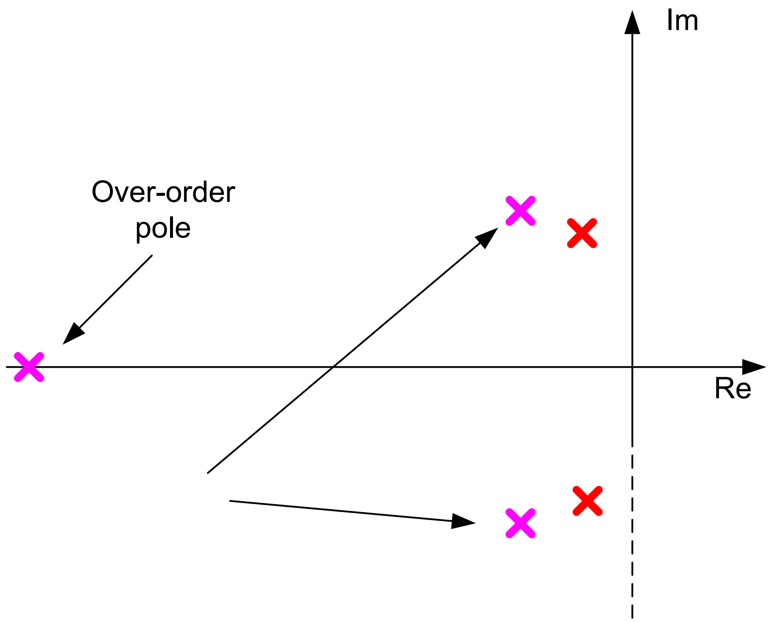


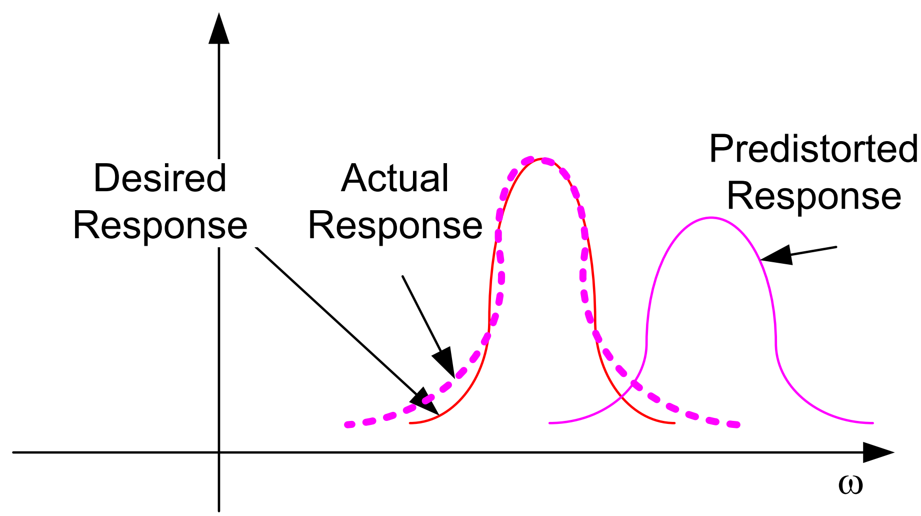
Pre-distortion concept

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained





Over-ordering Limitations with Pre-distortion

Parasitic Pole Affects Response

Predistortion almost always done even if benefits only modest

Not effective if significant deviations exist before predistortion

What can be done to address these problems?

2. Trimming

a) Functional Trimming

- trim parameters of actual filter based upon measurements
- difficult to implement in many structures
- manageable for cascaded biquads

b) Deterministic Trimming (much preferred)

- Trim component values to their ideal value
  - Continuous-trims of resistors possible in some special processes
  - Continuous-trim of capacitors is more challenging
  - Link trimming of Rs or Cs is possible with either metal or switches
- If all components are ideal, the filter should also be ideal
  - R-trimming algorithms easy to implement
  - Limited to unidirectional trim
  - Trim generally done at wafer level for laser trimming, package for link trims
- Filter shifts occur due to stress in packaging and heat cycling

c) Master-slave reference control (depends upon matching in a process)

- Can be implemented in discrete or integrated structures
- Master typically frequency or period referenced
- Most effective in integrated form since good matching possible
- Widely used in integrated form

Stay Safe and Stay Healthy !

End of Lecture 22